**Solutions**

**Tutorial 3**

**Week 4**

1. R revision – Data frames.
2. Consider the problems given in slides 37 and 46 in lecture notes for week 4.

The solutions are given in lecture slides. See sides 37 and 46.

1. You obtain the following estimates for an AR(2) model of some returns data.

where is a white noise error process. By examining the characteristic equation, check the estimated model for stationarity.

Solution:

The best way to check for stationarity is to express the model as a lag polynomial in *yt*.



Rewrite this as



We want to find the roots of the lag polynomial and determine whether they are greater than one in absolute value. It is easier (in my opinion) to rewrite this formula (by multiplying through by -1/0.682, using *z* for the characteristic equation and rearranging) as

*z*2 + 1.177 *z* - 1.466 = 0

Using the standard formula for obtaining the roots of a quadratic equation,

= 0.758 or 1.934

Since ALL the roots must be greater than one for the model to be stationary, we conclude that the estimated model is not stationary in this case.

1. (a) You obtain the following sample autocorrelations and partial autocorrelations for a sample of 100 observations from actual data:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Lag | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| SACF | 0.632 | 0.381 | 0.268 | 0.199 | 0.205 | 0.101 | 0.096 | 0.082 |
| SPACF | 0.420 | 0.104 | 0.032 | -0.206 | -0.138 | 0.042 | -0.018 | 0.074 |

Solution:

**(a)** We class an autocorrelation coefficient or partial autocorrelation coefficient as significant if it exceeds  = ± 0.196. Under this rule, the sample autocorrelation functions (sacfs) at lag 1 and 4 are significant, and the spacfs at lag 1, 2, 3, 4 and 5 are all significant.

This clearly looks like the data are consistent with a first-order moving average process since all but the first acfs are not significant (the significant lag 4 acf is a typical wrinkle that one might expect with real data and should probably be ignored), and the pacf has a slowly declining structure.

(b) Use the Ljung–Box Q∗ test to determine whether the first three autocorrelation coefficients taken together are jointly significantly different from zero.

Solution:

**(b)** The formula for the Ljung-Box Q\* test is given by

 ~ 

using the standard notation.

In this case, T=100, and m=3. The null and alternative hypotheses:

Test versus for some .

The test statistic is calculated as



The 5% and 1% critical values for a *χ*2 distribution with 3 degrees of freedom are 7.81 and 11.3 respectively. Clearly, then, we would reject the null hypothesis that the first three autocorrelation coefficients are jointly not significantly different from zero.